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Evolution of the Thickness Distribution of Ice in the Sea of Japan

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Abstract—Simple mathematical models of the thermodynamics of the ice-cover thickness and the thermodynamics of the area of an individual ice floe are proposed. The equations of the models allow for an explicit consideration of the spatial boundedness of the seawater region containing ice covers. A kinetic model of the evolution of the distributions of ice area and ice thickness is formulated on the basis of the gas-dynamic theory. Integration of the equations of this model over the surface areas of individual floes gives the thickness distribution of ice areas. Several special cases are studied analytically. The adequacy of the models is assessed. The results of simulations are presented.

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1. INTRODUCTION

Sea-ice covers represent multicomponent dynamic systems in which an external effect is determined by the air temperature and wind and sea-current characteristics. The models of ice-cover evolution are usually formulated for the long-term ice covers of the Arctic seas [1–5]. The adaptation of these models for seasonal ice covers is associated with a number of fundamental difficulties. For example, the differences of the objects lead to the absence, within the framework of the models, of the consideration of the evolution of ice covers during the period from their formation to their complete destruction. The importance of studying the ice cover of the Sea of Japan is due to the demand for making valid forecasts of its state to provide for the safety of the population in coastal regions, navigation safety, the complex of works aimed at development of the revealed gas–oil shelf fields, fishery, etc.

A specific feature of our model of ice-cover evolution lies in the fact that the model's equations allow explicit considerstion of the spatial boundedness of the seawater regions containing ice covers and the occurrence of a number of factors in the system that limit its evolution. On this basis, we formulate models of thermal evolution of the thickness and area for an individual floe. Then, we formulate the spatiotemporal model for the thickness distribution of ice. The parametric identification of the models and assessment of their adequacy is performed on the basis of statistical samples provided by V.V. Plotnikov and V.P. Tunegolovets.

2. INITIAL DATA

The basic data used by us characterize the 1961–1989 ice-cover parameters measured for ten-day time intervals over some seawater regions rather homogeneous in their ice conditions. In the northern part of the sea, 114 such regions were chosen. The entire seawater area was divided into elementary rectangular cells of 60 by 58 km, these sizes being determined by the size of the open-sea region (some coastal sea regions are partially occupied by dry land). For the year chosen, the ice-cover state is described by samples of ten-day distributions of ice compactness, ice-cover thickness, dominant size of floes, and ice-hummock age. The samples were formed on the basis of maps of ice-cover air reconnaissance and, data sets from coastal hydrometeorological stations and sites. Additional information obtained from radar and helicopter observations of ice, ship observations in passing, etc., was used. The fragments of the ice-condition maps for each region were used to construct mean ten-day maps for ten-day periods. These maps gave a possibility for estimating the averaged values of ice-cover characteristics, and these characteristics were related to the centers of the corresponding regions [6].

The temperature and wind regimes of air in the centers of the regions were specified by their temporal distributions with a step of one day at the standard over-ice horizons (2-m horizon for temperature and 10-m horizon for wind). The observations were performed from 1960 to 2001. The results of statistical analyses of the data averaged over the entire period of observations of the ice-cover regime of the Sea of Japan revealed a statistical coincidence between the ten-day air temperatures measured in the early periods of ice-cover formation and melting. For example, the mean air temperatures T_0 and T_{S1} for the ten-day periods antecedent to primary ice covering and to ice destruction (after the maximum compactness of ice), respectively, are as follows: $T_0 = -(8.4 \pm 4.2)$°C and $T_{S1} = -(9.1 \pm 4.7)$°C. Taking that ice melting begins in the ten-day period corresponding to the passage of the ice-cover thickness through its maximum, we obtain $T_{h1} = -(7.8 \pm 4.3)$°C (subscripts $S1$ and $h1$ are used to distinguish the parameters). According to the Student criterion, the parameters T_{S1} and T_{h1} are statistically indistinguishable.

3. THERMAL EVOLUTION OF ICE-COVER THICKNESS

Variations in the ice-cover thickness are determined by the heat fluxes onto the upper and lower ice-cover surfaces. The heat-balance equation can be written in the form

$$\rho\lambda\dot{h}_\Phi = q_I - q_{WI}, \tag{1}$$

where ρ, λ, and h are the ice density, latent melting heat, and thickness, respectively; $\dot{h} \equiv dh/dt$; q_I is the heat flux between lower and upper unit areas of the ice cover; and g_{WI} is the heat flux from water onto a unit area of the ice-cover lower surface. For the ice cover of the Sea of Japan, no significant mechanic variations in h occur [7]. Therefore, below, we will omit the subscript of the $\dot{h}$ symbol.

Taking that the temperature vertical profile in (1) is linear in the ice-cover thickness, i.e., $q_I = k(T_m - T)/h$, $q_{WI} = 0$, and that the snow and air temperatures are the same, we find that

$$h(t) = \gamma_h \left[\int_{t_0}^{t} (T_m - T)dt \right]^{1/2}, \tag{2}$$

where $\gamma_h \equiv (2k/\rho\lambda)^{1/2}$; k is the ice thermal conductivity; T_m is the temperature of the lower ice surface (following [8], $T_m = -1.8$°C); T is the air temperature; and t_0 is the instant of the primary ice-cover presence [9]. According to (2), at the final stage of the mature state of ice cover, $T_m - T \geq 0$ and, therefore, the thickness is not stabilized. For the Arctic ice cover, this feature could lead to overestimation of simulated thicknesses.

At the ice-cover formation–mature state stage, increases in the ice and snow thicknesses cause q_I to decrease and q_{WI} to increase primarily as a result of migration of a saline (liquid saturated with salts) from ice into water. Indeed, saline migration leads to salinization and, consequently, to densification of under-ice waters. This phenomenon initiates the convective mixing of water; i.e., cooled and salted waters are being steadily replaced by warmer deep waters, which increase the heat flux to the lower surface of ice. When q_I approaches the level of q_{WI}, the ice-cover thickness stabilizes.

The above considerations indicate that the difference $q_I - q_{WI}$ is determined by the air temperature, current thickness of the ice cover h, and maximum possible ice thickness H^* for a specific region of water area (for a shallow-water basin, H^* is equal to its depth; otherwise, the maximum value observed over a multiyear period is taken). It is clear that $q_I = q_{WI}$ in the absence of ice. At the stage of the primary formation of ice cover, when h is small, the difference $q_I - q_{WI}$ represents a nonnegative function of h. For mature ice covers, when the entire resource $(H^* - h)$ is "consumed," $q_I = q_{WI}$.

According to the aforementioned and to the concepts taken in [10–12] for resource–consumer systems, the approximation of $\dot{h}$ for the stage of ice-cover formation can be written in the form

$$\dot{h} = f_h(T)(H^* - h)h, \tag{3}$$

where $f_h(T) \geq 0$ is the relative h variation per unit time, a parameter that is related to the unit seawater "resource" available for ice formation [13, 14]. At the stage of ice-cover melting, the resource and its consumer change places; i.e., the upper quasi-homogeneous layer (UQL) is the resource for ice during ice-cover formation, and the ice cover is the resource for seawater during ice-cover melting. Then, $\dot{h}_W = \psi_h(T)(H^* - h_W)h_W$, where h_W is the thickness of the water layer in the upper sea thickness H^* and $\psi_h(T)$ is the relative variation of h_W per unit time, which is related to the unit ice resource available for water formation. After substitution of $h_W \equiv H^* - h$ into the equation for $\dot{h}_W$, we formally return to (3). However, now, we have $f_h(T) \leq 0$.

When the dynamic variables of environmental action (snow thickness, snowed-surface albedo) are given, a linear function of these variables can be used for approximation of f_h. For estimation of the corresponding coefficients, detailed long-term observational series should be used. However, observations of ice-cover in the Sea of Japan are fragmentary. Therefore, hereinafter, we consider a special case when an external action on the ice-cover thickness is determined by the air temperature at fixed values of the other dynamic characteristics. Under the assumption that the air temperatures during the initial ice-cover formation and during the initial ice-cover melting are the same, the $\dot{h}$ approximation can be written in the form

$$\begin{aligned} \dot{h}(t, T^*) = {} & (T^* - T)[\alpha_h\Theta(T^* - T) \\ & + \alpha'_h\Theta(T - T^*)](H^* - h)h, \end{aligned} \tag{4}$$

where T^* is the air temperature during the initial ice-cover formation; $\Theta(z)$ is the Heaviside function, which is equal to 1 if $z > 0$ and to 0 otherwise; and α_h and α'_h are nonnegative proportionality coefficients. The time step in the models under consideration is determined by the specified one-day temporal discreteness of air-temperature distributions. Their dimension is (m °C day)$^{-1}$; i.e., their numerical value determines the relative daily variation in the ice-cover thickness corresponding to air temperature variation by 1°C and related to the unit available resource of thickness. If the air temperature exceeds the T^* value, the sign of the right-hand side of Eq. (4) changes. The T^* value characterizes a definite combination of external conditions under which the initial ice formation and the initial ice melting begin. Evidently, T^* differs from the temperatures of the initial fast-ice formation and fast-ice melting (T_F^*). Indeed, in the coastal sea regions of fast-ice formation are relatively shallow and fresher as a result of their desalination by river waters and sinks of industrial plants. Therefore, ice formation in the open areas of the sea starts at atmospheric temperatures lower than it occurs in fast-ice regions. Ice-cover melting in the open areas of the sea begins at lower temperatures because the combined effect of solar radiation and warm sea currents is manifested actively in this period [15].

It is more demonstrative to compare models (2) and (4) under the condition that T = const. If $T < (aT^* - T_m)/(a-1)$, where $a = (4/27)H^{*3}\alpha_h/\mu$, the phase trajectories corresponding to (2) and (4) intersect on the phase plane $\{h, \dot{h}\}$ at two points (we denote the thicknesses at these two points as h_A and h_B, $h_A < h_B$). The intersection means that $(\alpha_h/\mu)(H^* - h)h^2 = (T_m - T)/(T^* - T)$. The inequality for T follows from an analysis showing that $(\alpha_h/\mu)(H^* - h)h^2$ is maximum at $h = (2/3)H^*$. It is clear that $0 < h_A < (2/3)H^*$ and $(2/3)H^* < h_B < H^*$. If t_A and t_B are the instants corresponding to h_A and h_B, respectively, the thicknesses calculated with $t \in [t_A, t_B]$ from (4) exceed those from (2). In the other cases, the opposite situation takes place.

The solution of (4) for the period of ice-cover formation is determined by the expression

$$h(t, T^*) = h_0 \times \frac{H^* \exp[\alpha_h H^*(T^* - \bar{T})(t - t_0)]}{H^* - h_0 + h_0 \exp[\alpha_h H^*(T^* - \bar{T})(t - t_0)]} - h_0,$$

where $\bar{T} = \frac{1}{t - t_0}\int_{t_0}^{t} T dt$ is the current mean temperature of air; $h_0 > 0$ is the thickness of the initial ice formation (primary floes); and $t_0 = t_0(x, y)$ is the instant of the initial ice formation (a function of geographic coordinates). For the melting stage, the substitution $\alpha_h \longrightarrow \alpha'_h$ is necessary. The presence of the integral over temperature allows a simple interpretation because this term reflects the inertia of the system in response to the thermal action of air. The function $h(t)$ has no inflection point if $-\dot{T}(t_C^{(h)})/[T^* - T(t_C^{(h)})]^2 > \alpha_h H^*/2$. This result corresponds to a rapid increase in the ice-cover thickness. Otherwise, the function $h(t)$ has an inflection point at $t = t_C^{(h)}$. Thus,

$$h(t_C^{(h)}, T^*) = \{H^* - \alpha_h^{-1}\dot{T}(t_C^{(h)})/[T^* - T(t_C^{(h)})]^2\}/2.$$

Then, there is a definite "incubation period" $t_C^{(h)}$ during which a decrease in T causes a thin ice cover to form. Under real conditions, this period is due to the following processes. At the preliminary stage of ice-cover evolution, isolated assemblages of ice germs appear. Then, they aggregate and stick together. After that, the seawater area of the region begins to be covered with a thin ice film whose thickness remains almost constant. Finally, fast thickening of this film proceeds up to its limit thickness.

4. KINETIC MODEL OF EVOLUTION OF ICE-COVER AREAS AND THICKNESSES

When formulating the model, we make the following assumptions. (i) Depending on T, floes with thickness h_0 form or disappear in a unit time. (ii) The ice-cover thickness is leveled by a diffusion mechanism. (iii) In coastal regions, some open-sea floes switch to fast ices during ice-cover formation, while the opposite situation occurs during ice-cover melting. (iv) Collisions between floes do not reduce their surfaces. The fulfillment of assumptions (i)–(iii) is due to natural temperature variations, the choice of the time step, and the presence of friction at the lower surface of ice cover. The fulfillment of assumption (iv) is due to the results of statistical analysis of the linear sizes of isolated floes [7, 13]. According to assumption (iv), the floe area a is an additive variable of the system: as a result of collisions of floes, a floe of the summarized area forms.

Under these assumptions, the evolution of the density of the distribution of the floe number $n = n(x_1, x_2, t, a, h)$ for the open portions of sea areas is determined from the balance relation

$$\partial n/\partial t + \partial(u_i n)/\partial x_i + \partial(\dot{h}n)/\partial h + \partial(\dot{a}n)/\partial a = f_{ah} + D\partial^2 n/\partial h^2 + \varphi + S + R, \quad (5)$$

where $u = (u_1, u_2)$ is the drift velocity of ice; $x = (x_1, x_2)$ are the spatial coordinates; $\dot{a} \equiv da/dt$ is the rate of the thermal growth of the area of an isolated floe; $f_{ah} \equiv f(a, h_0, T)$ is the rate of formation (melting) of floes of the thickness h_0 and area a; D is the diffusion coefficient, which is taken to be constant for simplicity; $\varphi_n = \varphi_n(T, n)$ characterizes the dependence of n on the fast-ice area ($\varphi_n \geq 0$ in the coastal seawater strips only); and the relations $S = S(x_1, x_2, a, h, T)$ and $R =$

$R(x_1, x_2, a, h, T)$ characterize the dynamics of aggregation and fragmentation of floes in a unit time.

In the practice of sea-ice research, the drift velocities of floes are calculated from the wind speed [1, 4, 16].

The model for $\dot{a}$ is formulated on the basis of the assumptions made in the model for $\dot{h}$. For a, the current resource is the free water area $A_W = A^* - A_1$ in the region under consideration, where A^* is the region area, $A_I \equiv \int_0^{H^*}\int_0^{A^*} andadh + A_I^{(F)}$ is the ice-cover area, and $A_I^{(F)}$ is the total area of fast ice. For the melting stage, the free water area should also be taken into account: because of its small reflectance, water accumulates solar radiation to a greater extent than ice. A definite portion of solar radiation is consumed for melting and thermal destruction of ice. The equation for $\dot{a}$ can be written in the form

$$\dot{a} = [\alpha_a\Theta(T^* - T) + \alpha_a'\Theta(T - T^*)](T^* - T) \times (A^* - A_I)a, \tag{6}$$

where α_a and α_a' are nonnegative proportionality coefficients.

It is reasonable to determine f_{ah} via a function decreasing rapidly with a, for example, from the hyperbola $C_f(T, h)(a_0 + a)^{-k}$, where the first factor differs from zero only at $h = h_0$, a_0 is the smallest area of an individual floe, and $k > 2$. The function $C_f(T, h)$ can be determined from the condition that $I = \int_0^{H^*}\int_0^{A^*} f_{ah}dadh$ is the number of floes formed or melted during ice-cover formation or melting. It is reasonable to assume that I is proportional to the resource available for ice formation $A^* - A_I$ in the first case and to the current area of thin ice $A_0 \equiv A(t, a, h_0)$ in the second case. In the two cases, the intensity of processes is determined by the difference $T^* - T$. Taking that $C_f \sim T^* - T$, we have $I = (T^* - T)[C(A^* - A)\Theta(T^* - T) + C'A_0\Theta(T - T^*)]$, where C and C' are nonnegative proportionality coefficients. Because $a_0 \ll A^*$, we have

$$C_f(T, h) = (T^* - T)[C(A^* - A_f)\Theta(T^* - T) + C'A_0\Theta(T - T^*)](k-1)a_0^{k-1}\delta_{h, h_0},$$

where δ_{h, h_0} is the Kronecker delta.

We take the following representation for φ_n:

$$\varphi_n(a, h) = -b_{ah}(T^* - T)n(a, h)\Theta(T^* - T) + [b_{T, n}(T - T_B^*) - b_{h, n}h]A^{(B)}(h)\Theta(T - T_F^*), \tag{7}$$

where b_{ah}, $b_{T, n}$, and $b_{h, n}$ are nonnegative proportionality coefficients and $A^{(F)}(h)$ is the area of fast ice with thickness h. On the right-hand side of (7), the first term characterizes the transformation of floes of the open coastal seawater regions into fast ices during ice-cover formation and the second term characterizes the transformation of fast-ice floes into ice floes of open seawater areas during ice-cover melting and thermal destruction.

A decrease in the air temperature leads to a rapid regelation of small ice pieces. Therefore, aggregation is significant only in the initial stage of ice-cover formation, when ice floes in the seawater area are so sparse that only their pair collisions should be taken into account and higher-order collisions can be disregarded. In this case, to write $S = S(a, h)$ for this step, the corresponding form of the so-called coagulation term in the Smoluchowski kinetic equation [17–19] can be used. Here, this term has the following form:

$$S(a, h) = \frac{1}{2}\iint_{\Omega ah} \beta(ah - a'h', a'h')n(a'', h'') \times n(a', h')da'dh' - n(a, h) \times \iint_{\Omega} \beta(ah, a'h')n(a', h')da'dh',$$

where $\Omega_{ah} = \{(a', h') : 0 < a' < a, 0 < h' \le h\}$; $\beta(z, y)$ is the kinetic-equation kernel, which represents a symmetric function; $a'' = a - a'$; and $h'' = (ah - a'h')/(a - a')$. This representation of $S(a, h)$ takes into account the fact that aggregation is usually formalized in terms of masses. We propose the following aggregation mechanism: the area and the thickness of an aggregated structure are equal to the sum of the areas of individual floes and to the ratio of the sum of the volumes of individual floes divided by their total area, respectively.

When writing R, we assume that each fragment of a crashed floe has the thickness of the initial floe. Then, the modification of the corresponding term in the Melzaka kinetic equation [19] takes the form

$$R(a, h) = \int_a^{A^*} \gamma(a', a, h)n(a', h)da' - a^{-1}n(a, h) \times \int_0^a \gamma(a, a', h)da',$$

where $\gamma(a', a, h)$ is the probability of formation of floes with the area a as a result of crushing the initial floe with an area $a' > a$. Normalization of $\gamma(a', a, h)$ should be performed in such a way that the integral $P(a, h) = a^{-1}\int_0^a \gamma(a, a', h)da'$ is equal to the probability of destruction of a floe with the area a.

Since consider the entire cycle of ice-cover evolution, the initial distribution is taken to be zero. It is clear that the boundary conditions for (7) follow from (4) and (6) and imply the absence of the corresponding fluxes at the edge boundaries of gradations of thicknesses and areas of floes.

5. EVOLUTION OF THE THICKNESS DISTRIBUTION OF ICE

The equation for the thickness distribution of ice $A = A(x_1, x_2, t, h)$ with consideration for (8) (see below) is obtained as a result of multiplication of (5) by a and subsequent integration of the resulting expression over this variable:

$$\partial A/\partial t + \partial u_i A/\partial x_i + \partial \dot{h} A/\partial h = (T^* - T) f_{Ah}(A) + D\partial^2 A/\partial h^2 + \varphi_A,$$

$$\partial A^{(F)}/\partial t + \partial \dot{h} A^{(F)}/\partial h = (T_F^* - T) f_{Ah}(A^{(F)}) + D\partial^2 A^{(F)}/\partial h^2 - \varphi_A,$$

$$f_{Ah}(A) = (\alpha_a A + \alpha_{ah}\delta_{h, h_0})(A^* - A_I)\Theta(T^* - T) + [\alpha'_a (A^* - A_I)A + \alpha'_{ah} A_0 \delta_{h, h_0}]\Theta(T - T^*), \quad (8)$$

$$\varphi_A = -b_{ah}(T^* - T)A\Theta(T^* - T) + [b_T(T - T_F^*) - b_h h]A^{(F)}\Theta(T - T_F^*),$$

where $A(t, h) = \int_0^{A^*} an(a, h)da$, $A^{(F)}(t, h) = \int_0^{A^*} an^{(F)}(a, h)da$; $A_I = \int_0^{H^*}(A + A^{(F)})\,dh$ $b_h = b_{h,n}A^{*2}/2$, and $b_T = b_{T,n}A^{*2}/2$. When writing the equation for the evolution of the areas of fast ices with different thicknesses $A^{(F)}$, we take into account the following. Formally, the fast ice can be described by the distribution density of the number $n^{(F)}(x_1, x_2, t, a, h)$ of floes, where the number of floes of each gradation (a, h) is 0 or 1. Then, (5) determines the evolution of $n^{(F)}(x_1, x_2, t, a, h)$ as well if the second term on the left-hand side of (5) is taken equal to zero and if $S \equiv 0$ (i.e., no aggregation occurs). When writing (8), we also take into account that $\int_0^{A^*} aS da = 0$ and $\int_0^{A^*} aR da = 0$; i.e., within the framework of an individual gradation of thickness, the redistribution of the areas of floes does not change their total area.

The initial and boundary conditions for (11) take the form

$$A(x_1, x_2, t_0, h) = A^{(F)}(x_1, x_2, t_0, h) = 0$$
$$\text{and} \quad \dot{h}A|_{h = h_0, H^*} = \dot{h}A^{(F)}|_{h = h_0, H^*} = 0. \quad (9)$$

We take the time step equal to one day. Observations and analysis of the orders of magnitudes of the terms in the corresponding equations show that, for this time step, the drift velocity of ice cover (but not of individual floes) is quasi-stationary in character and determined by simple relations [1, 4, 16].

When fast ice constitutes a major portion of ice at the ice formation–mature state stage, the dynamics of the total fast-ice area is described by the expression

$$A_I^{(F)}(t) = \alpha_{ah} h_0 A^* \times \frac{\exp[(\alpha_a A^* + \alpha_{ah} h_0)(T_F^* - \bar{T})(t - t_0)] - 1}{\alpha_a A^* + \alpha_{ah} h_0 \exp[(\alpha_a A^* + \alpha_{ah} h_0)(T_F^* - T)(t - t_0)]},$$

where $\bar{T} = \frac{1}{t - t_0}\int_{t_0}^{t} T\,dt$ is the current mean air temperature. The behavior of $A_I^{(F)}$ is similar to that of $h(t)$ from (4); namely, the occurrence (absence) of an inflection point is determined in both cases by the rate of air cooling. If $-\dot{T}(t_C^{(A)})/[T_F^* - T(t_C^{(A)})]^2 > \alpha_a A^* + \alpha_{ah} h_0$, no inflection point occurs in the function $A_I^{(F)}(t)$. This result corresponds to a rapid increase in the fast-ice area. Otherwise, the function $A_I^{(F)}(t)$ has an inflection point at $t = t_C^{(A)}$:

$$A_I^{(F)}(t_C^{(A)}) = \{(A^* - \alpha_a^{-1}\alpha_{ah} h_0) - \alpha_a^{-1}\dot{T}(t_C^{(A)})/[T_F^* - T(t_C^{(A)})]^2\}/2.$$

When fast ice occupies the entire region at the ice formation–mature state stage, the evolution of fast-ice areas with different thicknesses is determined by the expressions

$$A^{(F)}(t, h) = (H^*/h)^2 \sum_{k=1}^{K} B_k [h/(H^* - h)]^{1 + \lambda_k} \times \exp[-\lambda_k \alpha_h H^*(T_F^* - \bar{T})t], \quad h < H^*,$$

$$A^{(F)}(t, H^*) = A^* - \int_{h_0}^{H^* - \Delta h} A^{(F)}(t, h)dh,$$

where K is the number of thickness gradations; $\{\hat{A}_k\}$ should be chosen from the distribution of areas at the instant $t = t_f$, when the fast-ice area attains the entire area of the region; $\lambda_k > 0$ is a dimensionless parameter used for separation of variables; $\bar{T} = \frac{1}{t_g - t_f}\int_{t_f}^{t_g} T\,dt$ is the mean air temperature over the period $t_f < t \le t_g$, where t_g is the instant when water patches appear; and Δh is the step of thickness gradation. For simplicity, in these expressions, we disregard the processes of diffusive redistribution of ice-cover areas by their thicknesses (diffusion does not determine the character of the processes, but only smoothes the distribution). According to the above expressions, a decrease in the temperature leads to a decrease in the portion of thin ices and an increase in the portion of thick ices. During the period of ice-cover melting, the opposite situation occurs.

6. PARAMETRIC IDENTIFICATION AND SIMULATION EXPERIMENTS

For parametric identification of the model described by Eqs. (8) and (9), we use a sample averaged for the observation period and containing the areas of ice covers with different thicknesses. The element $A^{(D)}_{r,d,g}$ of the sample determines the area of the gth gradation of ice thickness in the dth ten-day period for the rth region (we consider six nonuniform gradations of thickness [6, 20]). The following expression corresponds to the element $A^{(D)}_{r,d,g}$: $A^{(M)}_{r,d,g}(\theta) = 0.1 \times \sum_{t=10d-9}^{10d} \sum_{j \in J(g)} [A(x_1, x_2, t, h_j, \theta) + A^{(F)}(x_{1r}, x_{2r}, t, h_j, \theta)]$, where the set of parameters is determined by $\theta = (\alpha_h, \alpha'_h, \alpha_a, \alpha'_a, \alpha_{ah}, \alpha'_{ah}, T^*_F, T^*, D, b_{ah}, b_T, b_h)$ and $J(g)$ are the numbers of the intervals of uniform fragmentation $(0, H^*]$ that cover the gth interval of thickness gradation.

To estimate the elements θ, we calculate the minimum of the functional

$$\Phi(\theta) = e^T(\theta)e(\theta),$$

where $e(\theta)$ is the column vector of residuals, whose μth element is $(A^{(D)}_{r,d,g} - A^{(M)}_{r,d,g}(\theta)$; $\mu = 6\{\sum_{r'=1}^{r-1}[d_1(r') - d_0(r')] + d - d_0(r) + r - 1\} + g$, $\mu = 1$–8022; $d_0(r)$ and $d_1(r')$ are the initial and final ten-day periods of evolution in the rth region; $r = 1$–114; and $g = 1$–6. To minimize $\Phi(\theta)$, the Gauss method [21] is used. To approximate the gradient, one-sided finite differences with the increment of the argument $\delta\theta_\alpha = 10^{-3}\theta_\alpha$ for $\Phi(\theta)$ are used.

The initial approximation for the solution of the problem on $\min\limits_{\theta}\Phi(\theta)$ is determined as follows. The approximate T^* is taken to be –8°C (in the Northern Hemisphere, its isotherm for the near-water air coincides with the boundary of sea ices [22]). The approximations for α_h, α'_h, α_a, α'_a, T^*_F, α_{ah}, and α'_{ah} were obtained on the basis of a sample of ice areas of different thicknesses in those coastal regions where the ice cover consists mainly of fast ices (in the water area of the Sea of Japan, 14 such regions occur). To calculate α_h and T^*_F, we used (4) and approximated the histograms of ten-day mean thicknesses by the expression $h_r(t) = \sum_{i=1}^{k_{r,h}} c_i^{(r,h)}(t - t_0^{(r)})^i$, where $t_0^{(r)}$ means the first day of ice occurrence in the water area of the rth region, $k_{r,h}$ is the polynomial degree, and t is the current day of the year. For each r, the solution of the problem $\min\limits_{\{c_i^{(r,h)}\}} \sum_{d=d_0(r)}^{d_1(r)} [h^{(D)}_{d,r} - 0.1\sum_{i=1}^{k_{r,h}} c_i^{(r,h)} \times \sum_{t=10d-9}^{10d}(t - t_0^{(r)})^i]^2$ can be obtained via the least-squares method ($k_{h,r}$ values were chosen through an interactive search). This method was also used to solve the problem $\min\limits_{(\alpha_h,\beta_h)} \sum_{r \in R_F} \sum_{t=10d_0(r)-9}^{10d_1(r)} \dot{h}_r/(H^* - h_r)h_r - \beta_h - \alpha_h T_r]^2$, where R_F are the numbers of the regions and $T_r = T_r(t)$ is the temperature of air over the ice cover. Thus, β_h/α_h is the estimate of T^*_F. The quantities α_a and α_{ah} were estimated in a similar way. Instead of (4), we used the result of integration of the second equation in (11) over h. In this case, α_a and α_{ah} is the solution of the problem $\min\limits_{(\alpha_a,\alpha_{ah})} \sum_{r \in R_F} \sum_{t=10d_0(r)-9}^{10d_1(r)} [\dot{A}_r/(A^*_r - A_r)(T^*_F - T_r) - \alpha_a A_r - \alpha_{ah}h_0]^2$, where $A_r(t) = \sum\limits_{i=1}^{k_{r,h}} c_i^{(r,A)}(t - t_0^{(r)})^i$ is the fast-ice area and A^*_r is the area of the seawater space of the rth region. The sets $\{c_i^{(r,A)}\}$ were calculated with the least-squares method on the basis of ten-day samples of the total fast-ice areas. To construct the initial approximation, we approximated b_{ah}, b_T, b_h, and D via an interactive procedure.

According to the simulation experiments, the estimated parameters are within the following limits: $\hat{T}^*_B = -(6.3 \pm 0.8)$°C, $\hat{T}^* = -(7.6 \pm 0.9)$°C, $\hat{\alpha}_h = (4.338 \pm 1.024) \times 10^{-4}$, $\hat{\alpha}'_h = (7.877 \pm 0.965) \times 10^{-4}$, $\hat{\alpha}_a A^* = 2.482 \pm 0.471) \times 10^{-2}$, $\hat{\alpha}'_a A^* = (2.580 \pm 0.586) \times 10^{-2}$; $\hat{\alpha}_{ah} A^* = (4.843 \pm 0.962) \times 10^{-2}$, $\hat{\alpha}'_{ah} A^* = (4.047 \pm 0.925) \times 10^{-2}$, $\hat{D} = (2.216 \pm 0.721) \times 10^{-3}$, $\hat{b}_{ah} = (6.931 \pm 1.096) \times 10^{-1}$, $\hat{b}_T = (2.744 \pm 1.046) \times 10^{-1}$, and $\hat{b}_h = (6.186 \pm 2.349) \times 10^{-1}$. The dimension of α_h and α'_h is (m °C day)$^{-1}$; the estimations of α_a, α'_a α_{ah}, and α'_{ah} are given on the scale of the area of the open-sea region and their dimension is (m^2 °C day)$^{-1}$; and the dimensions of D, b_{ah} and b_T, and b_h are m^2/day, (°C day)$^{-1}$, and (m day)$^{-1}$, respectively. According to these estimations, the measure of the intensity of the thermal destruction of the ice-cover thickness α'_h is almost twice as large as the measure of the intensity of formation of the ice-cover thickness α_h. This result agrees well with the assumptions made at the stage of formulation of (11).

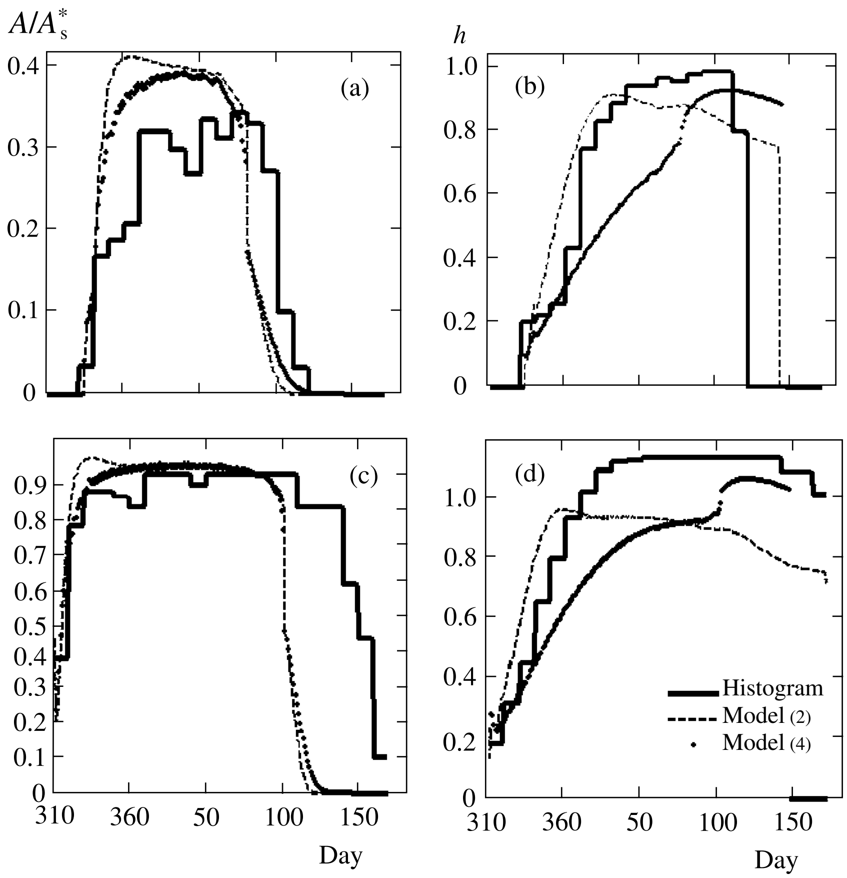

Fig. 1. Temporal variations in the area of ice cover and its mean thickness (a, b) in the Gulf of Peter the Great and (c, d) at the entry of the Tatar Strait (the abscissa axis is current days of the year).

The expression $\sum_{r \in R_F} \sum_{d = d_0(r)}^{d_1(r)} h_{d,r}^{(D)} \times \left[\int\limits_{10d_0(r)-9}^{10d} (T_m - T) \right]^{1/2} \Bigg/ \sum_{r \in R_F} \sum_{d = d_0(r)}^{d_1(r)} \left[\int\limits_{10d_0(r)-9}^{10d} (T_m - T) \right]^{1/2}$ is used for estimation of the parameter γ_h in model (2). In this case, $\hat{\mu} = (0.033 \pm 0.014)$ m (°C)$^{-1/2}$.

In Fig. 1, several curves characterizing temporal variations of the ice-cover area and mean thickness for two contrasting sea regions are presented. The ice-cover area is normalized by the open-sea area A_S^*. Figures 1a and 1b and Figs. 1c and 1d characterize the ice conditions in the Gulf of Peter the Great and at the entry of the Tatar Strait, respectively. Analysis of the curves shows that the model agrees well with the measured data. This conclusion follows directly from analyses of the corresponding correlations, which can be used as a measure of adequacy of the model for the real observational data [21]. Here, the correlations exceed 0.729.

The dashed lines in Fig. 1 show the simulation results obtained with the use of (2) as the ice-cover thickness. The points marked with symbols are obtained with the use of (4) for specification of h. The curves relating to the ice-cover areas are almost coincident. The present and earlier comparisons of the curves characterizing the thicknesses lead to similar results: during the time interval from formation to the mature state of the ice cover, the dashed lines in Figs. 1b and 1d are located higher than the lines corresponding to (4). At the final stage, the opposite situation takes place. It is also necessary to note the observed fact that, at the final stage of evolution, when the ice-cover area decreases, the mean ice-cover thickness is, for a time, a nondecreasing function. The cause of this situation is that the relative rate of decreasing of the ice-cover volume $(-\dot{V}_I/V_I)$ does not exceed the relative rate of decreasing in the ice-cover area $(-\dot{A}_I/A_I)$. Therefore, one or two ten-day periods before full ice-cover destruction, rather thick ice structures of small areas occur in the seawater space of the regions under study. Then, the thickness of these ice structures decreases rapidly to zero. Such variations in the ice-cover thickness correspond to a greater extent to model (4) than to model (2). Therefore, it is reasonable to use models (2) and (4) for description of ice-cover formation and ice-cover destruction, respectively.

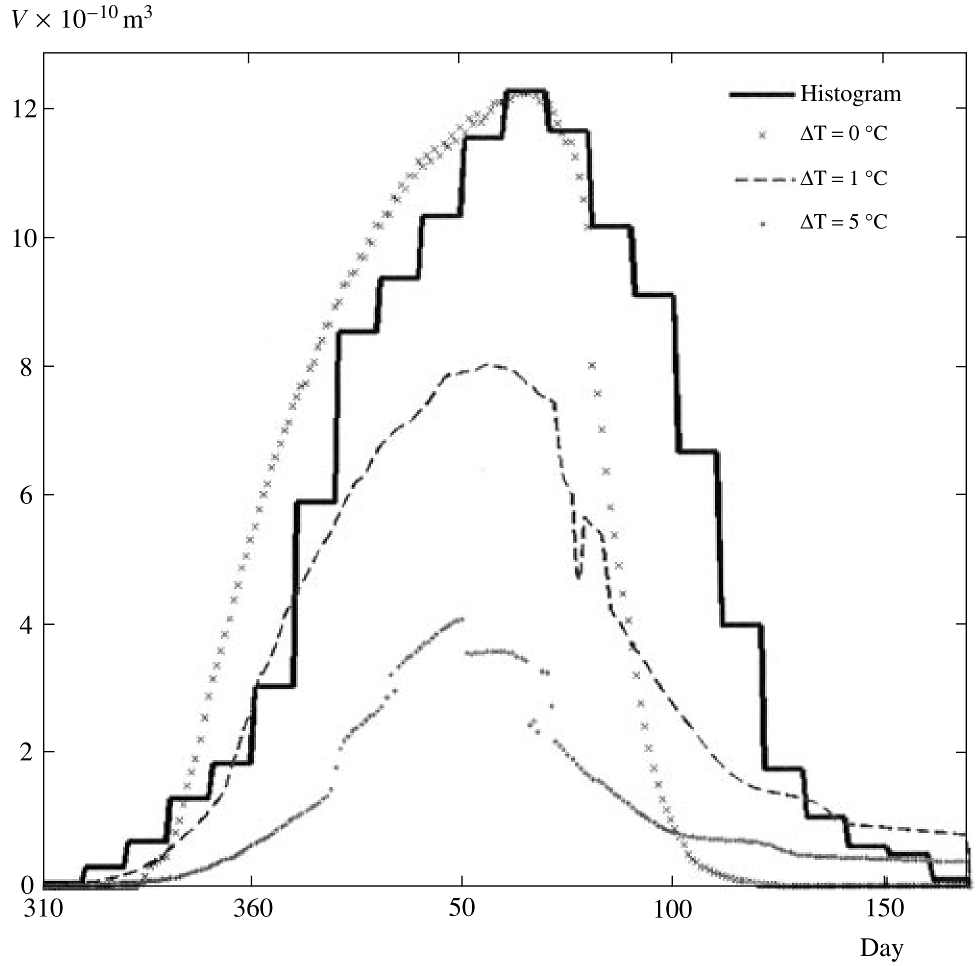

Fig. 2. Dynamics of the ice volumes of the Sea of Japan during variations in the atmospheric temperature.

Model (8), (9) can be used for different prognostic experiments, in particular, for estimating the possible effect of climate changes on the ice cover of the Sea of Japan. An analysis of the ice-thickness measurements performed from American and British submarines at 29 sites in the Arctic Ocean showed that the mean ice thickness decreased by about 43% for the period 1958–1997 [5, 11]. For this period, the mean ice-cover thickness decreased approximately from 3.5 to 2 m. For the ice cover of the Sea of Japan, there is no information of such a kind. This problem is of interest for many reasons. First, quantitative estimates of the effect of industrial and manufacturing plants on the environment are necessary for ecological revision. Second, such information is useful for complex estimation of the possible state of the climatic system in the future.

In these simulations, the wind regime in the 2-m-thick air layer over the ice cover was taken to be similar to that taken earlier for assessment of the model's adequacy. The atmospheric temperature was taken to increase with a step of 0.5°C. The results of simulation are given in Fig. 2 in the form of plots characterizing variations in the total volumes of ice. The scale of ice-volume measurements is -10^{10} m^3. According to these results, an increase in the temperature by 1 or 1°C leads to losses in the total ice volume of 1.695×10^2 m^3 (19% of the current ice volume) or 6.092×10^{12} m^3 (69% of the current ice volume).

In order to reveal the order of magnitude of these numbers, they should be expressed in terms of the areas of open-sea regions. Then, in the first case, the total ice losses constitute the ice cover of almost 249 such regions with 1-m-thick ice cover and, in the second case, this number corresponds to the area of 826 regions.

It is clear that the results obtained give only a rough idea of the possible consequences of climate changes. The point is that global warming leads to the intensification of precipitation and, as a consequence, to a change in the salt regime of the ocean. As a result, atmospheric conditions and the atmospheric temperature at which the primary ice cover forms change also. Different trends disregarded in our model are also possible. Nevertheless, the present concepts allow the assessment of the consequences of climate changes.

7. CONCLUSIONS

Our study allows us to make the following conclusions.

(i) The thermal evolution of the ice cover of the Sea of Japan can be formalized on the basis of concepts of the "resource–consumer" system, in which the intensity of interaction is characterized by the air temperature. These concepts allow the formulation of models for the thermal evolution of the thickness of ice and for the thermal evolution of the area of an individual floe. Within the framework of these models, the observed peculiar features of sea-ice evolution, in particular, the effect of the rate of air cooling on the character of ice-cover formation, can be explained.

(ii) On the basis of the gas-dynamic approach, a model has been formulated for the evolution of ice areas and thicknesses, has been further used to formalize the evolutionary model for the area distribution of ice with different thicknesses.

(iii) The method of parametric identification of this model has been developed, and its adequacy for the initial data has been assessed.

(iv) The effect of possible climate changes on the ice cover of the Sea of Japan has been estimated.

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